NOTATION

dA_i, an element of the i-th surface; $dE(\bar{r}_{dA_i})$, flux density in the vicinity of the point $\bar{r} \in dA_i$; R_1 and R_2, principal radii of curvature of the reflected front; $K = 1/R_1R_2$, total (Gaussian) curvature of the reflected front; $H = 0.5((1/R_1) + (1/R_2))$, average curvature of the reflected front; ρ, reflection coefficient of the specular surface.

LITERATURE CITED

1. A. V. Arendarchuk, Elektrotekh. Promst., Ser. Elektrotermiya, No. 9 (259), 2-3 (1984).
2. Yu. A. Surinov, Heat Transfer and Thermal Modeling [in Russian], Moscow (1959), pp. 319-349.
3. A. V. Arendarchuk, Inzh.-Fiz. Zh., 50, No. 3, 494 (1986).
4. M. Born and E. Wolf, Principles of Optics, 4th ed., Pergamon Press, Oxford—New York (1969).
5. O. N. Stavroudis, The Optics of Rays, Wavefronts and Caustics, London (1972).
6. Yu. A. Kravtsov and Yu. I. Orlov, Geometrical Optics of Nonuniform Media [in Russian], Moscow (1980).
7. V. A. Fok, Problems of Diffraction and Propagation of Electromagnetic Waves [in Russian], Sovetskoe Radio, Moscow (1970), pp. 155-178.
8. I. M. Fuks, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 8, No. 6, 1078-1088 (1965).
9. F. G. Bass and I. M. Fuks, Wave Scattering on a Statistically Uneven Surface [in Russian], Sec. 28 (1972), pp. 313-319.
10. D. G. Burkhard, D. L. Shealy, and R. U. Sexl, Int. J. Heat Mass Transfer, 16, 271-280 (1973).
11. D. G. Burkhard and D. L. Shealy, J. Opt. Soc. Am., 63, No. 3, 299-304 (1973).
12. D. L. Shealy and D. G. Burkhard, Opt. Acta, 20, No. 4, 287-301 (1973).
13. A. T. Vlasov, I. A. Vatutin, and I. V. Skutova, Processes of Heat and Mass Exchange in Elements of Thermooptical Devices [in Russian], Minsk (1979), pp. 46-49.
14. D. L. Shealy and D. G. Burkhard, Opt. Acta, 22, No. 6, 485-501 (1975).
15. B. Ya. Gel'chinskii, Dokl. Akad. Nauk SSSR, 118, No. 3, 458-460 (1958).
16. G. L. Voronkov, Svetotekhnika, No. 1, 19-20 (1973).
17. N. N. Ermolinskii and V. D. Komissarov, Byull. Vses. Elektrotekh. Inst., No. 2, 21-26 (1934).
18. D. G. Burkhard and D. L. Shealy, Int. J. Heat Mass Transfer, 16, 1492-1496 (1973).
19. J. A. Plamondon and T. E. Horton, Int. J. Heat Mass Transfer, 10, 665-679 (1967).
20. J. B. Scarborough, Appl. Opt., 3, No. 12, 1445-1446 (1964).
21. H. J. Stalzer, Appl. Opt., 4, No. 9, 1205-1206 (1965).
22. A. Sonnefeld, Concave Mirrors [Russian translation], V. Fabrikant (ed.), Moscow—Leningrad (1935).

NONLINEAR EFFECTS DURING FILTRATION IN BEDS WITH LARGE-SCALE STRUCTURAL INHOMOGENEITY

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A model of the nonlinear hydraulic relation between the porous volumes of a significantly inhomogeneous bed is proposed, and its influence on the draining of unit volume of borehole, as well as on the curves of bed-pressure recovery and on the indicator diagrams of the borehole, is investigated.

Recently, the industrial petroleum content of distinctive clay bituminous rocks with unusual (in comparison with well-known collectors) and in many respects unique properties has been established; the collector of the so-called Bazhenov formation of western Siberia

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serves as the clearest example, and is exceptionally important in applied terms [1, 2]. Traditional methods cannot be applied to the rock of the Bazhenov formation: the well-known geophysical and hydrodynamic methods, which are highly recommended for other deposits, do not "work" in this case, or lead to very contradictory results. This seriously hinders calculations of the available reserves of petroleum, estimation of the productive characteristics of the collector, and the development of strategies and tactics on the more rational exploitation of deposits.

There have been many attempts to construct a well-founded structural—hydrodynamic model of these collectors; indicative examples may be found in [3-12]. These attempts are based on concepts, in various combinations, regarding infinite, closed, or quasi-closed petroleum-containing volumes with different degrees of inhomogeneity, regarding the presence or absence of additional petroleum sources in the volumes separating the cracks or highly porous layers or in the adjacent beds, regarding the strong or weak compressibility of a porous body in the extraction of petroleum, etc. As shown by critical analysis in [12], the full total of the geological and hydrodynamic information which has been accumulated cannot be explained by models based on hypotheses regarding a homogeneous infinite or closed bed, regarding a system of finite saturated volumes with a constant hydraulic interrelation, and regarding an infinite cracked-porous bed with double porosity and permeability. The so-called "quasiclosed elastoplastic" model, in which clear account is taken of the increase in filtration region (inclusion of new sections of the collector in the process) with exhaustion of the deposit and deformation of the collector with drop in effective bed pressure, is preferred [5].

Formulation of the Model

The most significant features of petroleum extraction from beds of the Bazhenov formation are as follows [1-12]. In the initial period of borehole operation, the face pressure sharply falls (as in a closed finite bed), but after some time the rate of this drop is much reduced (as if additional petroleum reservoirs have come into action), while in a series of cases pulsating conditions of borehole operation are observed. When the borehole is switched off, the characteristic time for pressure recovery is many times larger than that for homogeneous beds, although usually the pressure is not restored to the initial level of the bed pressure. On the indicator diagrams of the boreholes, there appear sections which are convex to the pressure axis, as if additional sections are included in the drainage region of the collector some time after the onset of borehole operation. Finally, the yields of even close-lying boreholes vary over broad limits, very strong differences in the composition of petroleum from these boreholes are noted, and the effective values of the bed pressure calculated from their operational indices also differ considerably. All this indicates the presence of significant large-scale inhomogeneity of collectors of Bazhenov type, which is also confirmed by the results of hydroprobing and a series of independent purely geological data.

According to the most reliable information and hypotheses (see [1, 2, 5, 12], for example), the collector of the Bazhenov formation at the large-scale hierarchical level is a set of individual porous and permeable volumes ("lenses") with linear dimensions from hundreds of meters to a few kilometers distributed in a consolidated weakly permeable matrix. It is possible to establish a hydraulic relation through the matrix between individual lenses under definite conditions; this relation is significantly nonlinear and, in any case, cannot be regarded as constant or weakly varying in the course of filtration (in contrast, for example, to the models in [10, 11]). The interaction of individual lenses ultimately depends on the features of their mutual position, their form, the position of the boreholes within them, and many other random factors, which cannot be taken into account within the framework of a single determinate theory. Therefore, some mean axisymmetric picture corresponding to a single plane lens of circular form with a borehole at its center is considered below; inflow of liquid from the surrounding medium, modeling "inrushes" of petroleum from neighboring lenses, is possible at the external boundary of the lens under appropriate conditions. Note that expansion of the filtration region over time was considered earlier in [5] on the basis of the formal assumption that the rates of this expansion and the increase in porosity of the collector with drop in pressure are proportional.

The appearance of flow between the lenses may be associated with two basic factors (see [12], for example). First, when the pressure drop between them reaches some critical value, hydraulic breakdown of the matrix rock separating the lenses is possible. Second, the flow in a weakly permeable clay matrix must be plastic in character, i.e., only begins when the pressure gradient reaches a critical limiting value, which leads to some global effect [13]. Therefore, even in accordance with the obvious generalization of the well-known Bingham model, it is supposed that flow from the surrounding medium into the given lens appears in the case when the pressure p_b at its external boundary is less than the critical value $p_* < p_\infty$. This flow ceases when p_b reaches a second critical value p_{**}, where $p_\infty > p_{**} > p_*$. This inequality means that, for crack opening (formation) or for the onset of motion in porous capillaries, more considerable forces are usually required than for maintenance of motion in already-existing cracks or in capillaries. The mechanism of "switching on and off" the motion is analogous to the well-known principle of "dry" friction, when the static force of friction is more than the dynamic force.

So as to be specific and simple, the porous material of the lens is assumed to be homogeneous and characterized by coefficients which are not dependent on the pressure of the fluid: the permeability k, piezoconductivity κ, and hydroconductivity $j = kb/\mu$. Considering only linear (conforming to Darcy's law) filtration in elastic conditions, the equation for pressure in conditions of axial symmetry is written

$$\frac{\partial p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right), \tag{1}$$

where the dimensionless time and radial coordinate, with scales R^2/κ and R, respectively, are introduced. At the borehole ($r = \varepsilon$), the boundary condition of constant yield or face pressure is given. When $\varepsilon \ll 1$

$$\lim_{r\to 0}\left(r\frac{\partial p}{\partial r}\right) = H \quad \text{or} \quad p|_{r=\varepsilon} = p_0. \tag{2}$$

At the external boundary of the lens ($r = 1$), the switching condition is written in the following form. For transition from the state of rest at this boundary to the state of motion through it with reduction in p_b

$$r\frac{\partial p}{\partial r}\bigg|_{r=1} = \begin{cases} Q + \beta(p_* - p_b), & p_b \leqslant p_*, \\ 0, & p_b > p_*. \end{cases} \tag{3}$$

For the cessation of an initially-existing flow through the lens boundary with increase in pressure p_b there, the switching condition takes the form in Eq. (3) but with replacement of p_* in Eq. (3) by p_{**}. Thus, the reduced flux through the lens boundary appearing in Eq. (3) increases discontinuously from zero to Q when it develops, and changes from $Q - \beta(p_{**} - p_*)$ to zero on disappearance. The quantities Q and β and also p_* and p_{**} must be regarded as characteristics of the collector which are specified a priori. The quantities H in Eq. (2) and Q in Eq. (3) are proportional to the borehole yield and the volume flux through the lens boundary, with coefficients inverse to j.

The initial conditions on the pressure field depend on the specifics of the particular problems and are formulated separately for different problems.

The boundary conditions of switching are significantly nonlinear, but the nonlinear problems which arise for Eq. (1) are easily reduced to a system of linear boundary problems in limited time intervals, which may be solved, for example, using the standard Fourier method of variable separation. Such solutions allow as high an accuracy as is desired to be obtained, but are very cumbersome, since they entail calculating the roots of transcendental equations and summing sufficiently complex series of Bessel functions, and therefore they are unfavorable even in computer calculations, requiring large amounts of processor time. Therefore, simple but sufficiently effective approximate solutions obtained by the method of integral relations are used below [14]. The accuracy of these solutions is completely satisfactory. Thus, in the problems below regarding borehole use with constant yield or face pressure (also solved by the Fourier method), the maximum errors of the integral-relation method are no greater than 2 and 10%, respectively.

<u>Problem of Borehole Startup</u>

As the initial condition in the given case, it is natural to take

$$p|_{t=0} = p_\infty, \tag{4}$$

which corresponds to an initially unperturbed lens; the solution of the problem in Eqs. (1)-(4) is sought in the form

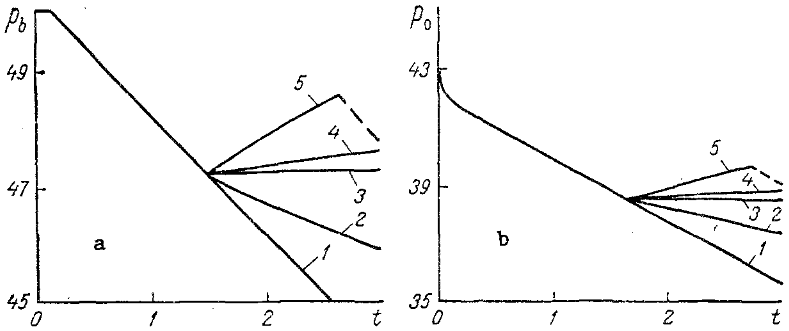

Fig. 1. Variation in pressure at lens boundary (a) and at face (b) in borehole operation with constant yield ($p_* = 47$, $p_{**} = 48.5$, $p_\infty = 50$ MPa, $\beta = 0.1$, $H = 1.0$ MPa, borehole radius 0.15 m, lens radius 700 m): 1) Q = 0; 2) 0.5; 3) 1.0; 4) 1.1; 5) 1.6 MPa; the dashed line corresponds to the onset of the development of self-oscillation; p_b, p_0, MPa.

$$p(t, r) = \begin{cases} p_1(t, r), & t \leqslant T, \\ p_1(t, r) + p_2(t - T, r), & t > T, \end{cases} \tag{5}$$

where T is the time of appearance of a liquid influx from the surrounding medium. The function p_1 in Eq. (5) describes the perturbing influence of a sink at the coordinate origin on the pressure field, while p_2 describes the influence of influx through the lens boundary. On the basis of Eqs. (2)-(4), it is simple to write the boundary and initial conditions which must be imposed on these functions.

Within the framework of the integral-relations method, it is assumed that

$$p_1(t, r) = p_\infty + \begin{cases} 0, & r \geqslant R_1(t) \\ u_1(t, r), & r < R_1(t) \end{cases} \Bigg| \; 0 \leqslant t \leqslant \tau_1 < T, \\ \quad u_2(t - \tau_1, r), \quad t > \tau_1; \tag{6}$$

$$p_2(t, r) = \begin{cases} 0, & r \leqslant R_2(t) \\ u_3(t, r), & r > R_2(t) \end{cases} \Bigg| \; 0 \leqslant t \leqslant \tau_2, \\ \quad u_4(t - \tau_2, r), \quad t > \tau_2. \tag{7}$$

Here R_1 and R_2 are dimensionless coordinates determining the boundaries of the corresponding "influence" regions; τ_1 and τ_2 are dimensionless times of propagation of the perturbation from the borehole to the external lens boundary and back. Choosing the trial functions in the necessary manner and requiring that the boundary conditions and the material-balance equation be satisfied, the integral-relation method [14] gives the following result for a borehole with fixed yield

$$\begin{gathered} u_1(t, r) = H \ln \frac{r}{R_1} - \frac{H}{2}\left[\left(\frac{r}{R_1}\right)^2 - 1\right], \\ u_2(t, r) = H \ln r - (H/2)(r^2 - 1) - 2Ht, \\ u_3(t, r) = \frac{(Q + 2\beta H t)(r - R_2)^2}{(1 - R_2)[2 + \beta(1 - R_2)]}, \\ u_4(t, r) = \left(1 - \frac{\beta}{2 + \beta} r^2\right) F(t) + \left[Q + 2\beta H (t + \tau_2)\right] \frac{r^2}{2 + \beta}, \end{gathered} \tag{8}$$

where

$$R_1(t) = \sqrt{8t}, \quad \tau_1 = 1/8, \quad T = 1/8 + (2H)^{-1}(p_\infty - p_*),$$

$$F(t) = \begin{cases} 2Qt, & \beta = 0, \\ 2Ht + \dfrac{Q - H}{\beta}\left(1 - \exp\dfrac{-8\beta t}{4 + \beta}\right), & \beta \neq 0, \end{cases} \tag{9}$$

$$\frac{dt}{df}=\frac{(Q+2\beta Ht)[(8-3f)f-\beta f^2(4-f)(2+\beta f)^{-1}}{24(Q+2\beta Ht)-2\beta Hf^2(4-f)},$$
$$R_2(t)=1-f(t).$$

If $\beta = 0$, the last equation is integrated in analytical form. With the obvious initial condition $t = 0$ when $f = 0$, it follows that $t = f^2/6 - f^3/24$, and hence

$$f=\frac{4}{3}\left\{1-2\cos\left[\frac{1}{3}\arccos\left(1-\frac{81}{16}t\right)+\frac{\pi}{3}\right]\right\},\quad \tau_2=\frac{1}{8} \tag{10}$$

where the expression for τ_2 follows from the condition $R_2(\tau_2) = 0$. When $\beta \neq 0$, the last equation in Eq. (9) is integrated numerically.

The solution of the analogous problem for a borehole with a fixed face pressure takes the form in Eqs. (5)-(7), as before, but the expressions for $u_i(t, r)$ are considerably more cumbersome. They are given here only for the case when $\beta = 0$, retaining terms of the order of $\ln \varepsilon$ and unity (inclusive). We have

$$\begin{gathered}u_1(t,\ r)=-(p_\infty-p_0)\left(\ln\frac{1}{\varepsilon}-1\right)^{-1}\left(\ln\frac{R_1}{r}-\frac{R_1-r}{R_1}\right),\\ u_2(t,\ r)=p_0+(p_\infty-p_0)\left(\ln\frac{1}{\varepsilon}-1\right)^{-1}\left(\ln\frac{r}{\varepsilon}-r\right)\exp(-\lambda t),\\ u_3(t,\ r)=Q(r-R_2),\\ u_4(t,\ r)=Q\left[\ln\frac{r}{\varepsilon}-\left(\ln\frac{r}{\varepsilon}-r\right)\exp(-\lambda t)\right],\end{gathered} \tag{11}$$

where

$$\begin{gathered}R_1(t)=(18t)^{1/3},\quad \tau_1=1/18,\\ T=\frac{1}{18}-\frac{1}{\lambda}\ln\left(1-\frac{p_\infty-p_*}{p_\infty-p_0}\right),\quad \lambda=2\left(\ln\frac{1}{\varepsilon}-\frac{7}{6}\right)^{-1},\\ R_2(t)=2\cos\left[\frac{1}{3}\arccos(1-3t)+\frac{\pi}{3}\right],\quad \tau_2=\frac{1}{3}.\end{gathered} \tag{12}$$

In both the cases here considered, the borehole operation has an influence on pressure p_b over the dimensionless time τ_1 after the onset of the process; the influx from the surrounding medium begins after a time T, and the borehole is sensitive to the influence of this influx up to a time $T + \tau_2$.

The variation in the pressure at the face and the external boundary of the lens is shown in Fig. 1 for a process with fixed yield. Two versions of development of the process are possible. If Q from Eq. (3) is no greater than the critical value Q_* which depends on β and H from Eq. (2) (note that Q and H have the dimensionality of pressure and β is dimensionless), stabilization occurs. In this case, p_b is in range (p_*, p_{**}) and

$$\lim_{t\to\infty}p(t,\ r)=p_*+H\left(\ln r+\frac{1}{2}-2\tau_2\right)+H\left(\frac{1+\beta\tau_2}{2+\beta}-\frac{1}{2}\right)r^2+\frac{Q-H}{\beta}. \tag{13}$$

When $Q = Q_*$, the function in Eq. (13) takes the value p_{**} at the point $r = 1$. If $Q > Q_*$, such steady asymptotic conditions are impossible. When p_b reaches the value p_{**}, the flux through the lens boundary ceases, p_b begins to fall, and when p_b reaches p_*, this flux begins again. This pattern corresponds to the establishment of self-oscillatory filtration conditions. The condition of onset of such filtration conditions is easily obtained from Eq. (13):

$$Q>Q_*=H\left[1-\frac{\beta(1-4\tau_2-\beta\tau_2)}{2+\beta}\right]+\beta(p_{**}-p_*). \tag{14}$$

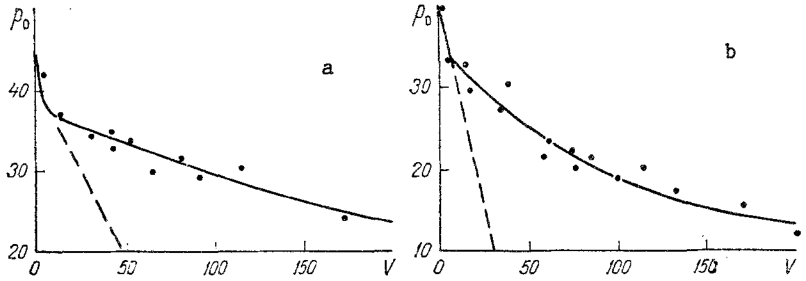

Fig. 2. Dependence of the face pressure on the total volume of petroleum taken in conditions of constant yield for borehole 27 (a) and 28 (b) of the Salymsk deposit; points correspond to experiment [15] and curves to theory; a) $2\pi \cdot jQ = 1.53 \cdot 10^{-3}$ m³/sec, $\beta = 0.01$, $j = 3 \cdot 10^{-10}$ m³/Pa·sec, $p_* = 46.15$ MPa; b) $2\pi jQ = 1.27 \cdot 10^{-3}$ m³/sec, $\beta = 0.008$, $j = 3.47 \cdot 10^{-10}$ m³/Pa·sec, $p_* = 39.1$ MPa; lens radius 1560 (a) and 910 (b) m; dashed curves correspond to the pressure drop in closed lenses ($Q = 0$, $\beta = 0$); V, 10^3 m³.

If $\beta = 0$ and $\tau_2 = 1/8$ in accordance with Eq. (10), the simple condition $Q > H$ follows from Eq. (14). For a process with fixed pressure at the borehole face, the condition of onset of self-oscillation is

$$Q > Q_* = (p_{**} - p_0)\ln^{-1}(1/\varepsilon) \quad (\beta = 0,\ p_* > p_0). \tag{15}$$

If this condition is not satisfied, stabilization occurs, with an asymptotic pressure profile $p_* + Q\ln(r/\varepsilon)$ and a dimensional borehole yield of $2\pi jQ$.

Steady Self-Oscillation

Consider a periodic process of filtration over a sufficiently large time from borehole startup, when the initial conditions cease to influence the characteristics of the process. Such steady self-oscillation arises when Eq. (14) or Eq. (15) holds and is evidently relaxational in character. Choosing the time origin at the moment when influx through the lens boundary ceases, and denoting the dimensionless lengths of the cycles of absence and presence of such influx by T_1 and T_2, so that $T_1 + T_2$ is the dimensionless period of self-oscillation, it is assumed that

$$p(t, r) = \begin{cases} p_1(\tau, r), & 0 \leqslant \tau < T_1, \\ p_2(\tau, r), & T_1 \leqslant \tau < T_1 + T_2, \end{cases} \tag{16}$$
$$\tau = t - [t(T_1 + T_2)^{-1}](T_1 + T_2),$$

where $[x]$ denotes the integer part of x. The function $p_i(t, r)$ is the solution of Eq. (1) with the boundary conditions at the borehole in Eq. (2) and the following boundary conditions at the external lens boundary

$$\frac{\partial p_1}{\partial r} = 0, \quad \frac{\partial p_2}{\partial r} = Q + \beta(p_* - p_2), \quad r = 1. \tag{17}$$

The initial conditions take the form of periodicity conditions

$$\begin{aligned} &p_1(T_1, r) = p_2(0, r), \quad p_2(0, 1) = p_*, \\ &p_2(T_2, r) = p_1(0, r), \quad p_1(0, 1) = p_{**}. \end{aligned} \tag{18}$$

The solution of this problem by the integral-relation method does not involve any fundamental difficulties. The final results are given here for $\beta = 0$, i.e., for an influx through the lens boundary which does not depend on p_b in the interval (p_*, p_{**}). For a borehole with constant yield

$$p_i(t, r) = \begin{cases} u_i(t + T_{3-i} - \tau_2, r), \ 0 \approx \varepsilon \leqslant r \leqslant R_2(t) \\ u_i(t + T_{3-i} - \tau_2, r) + (-1)^i u_3(t, r), \ r > R_2(t) \end{cases} \Bigg| \ t \leqslant \tau_2 \leqslant T_i, \tag{19}$$
$$u_{3-i}(t - \tau_2, r), \ t > \tau_2,$$

where

$$u_i(t, r) = H\left[\ln r - \frac{r^2-1}{2} - 2(t - T_{3-i} + \tau_2)\right] + v_i(t, r), \quad i = 1, 2;$$

$$v_1(t, r) = p_{**} + Q\left[\frac{r^2-1}{2} + 2(t - T_2 + \tau_2)\right]; \quad v_2(t, r) = p_*; \tag{20}$$

$$u_3(t, r) = \frac{Q(r-R_2)^2}{2(1-R_2)}; \quad R_2(t) = 1 - f(t),$$

and f and τ_2 are defined in Eq. (10).

The definitions of T_1 and T_2 here are

$$T_1 = (2H)^{-1}(p_{**} - p_* - Q/4), \quad T_2 = [2(Q-H)]^{-1}(p_{**} - p_* - Q/4). \tag{21}$$

The maximum p^+ and minimum p^- pressure at the face is

$$p^+ = p_{**} + 0{,}25(H-Q) + H\ln\varepsilon, \quad p^- = p_* + 0{,}25H + H\ln\varepsilon. \tag{22}$$

Thus, the amplitude $p^+ - p^- = p_{**} - p_* - 0.25Q$ of pressure oscillations at the face is somewhat smaller than the amplitude of pressure oscillations at th lens boundary. Note that T_1, T_2, p^+, and p^- may be determined experimentally from the curve of face-pressure variation. Then it is simple to find Q, p_*, and p_{**} – i.e., quantities characterizing the hydraulic relation between the lenses in the inhomogeneous bed – from Eqs. (21) and (22).

For a borehole with constant face pressure, the solution takes the form in Eq. (19), as before, but Eq. (20) is replaced by the relations

$$u_i(t, r) = p_0 + C_i\left(\ln\frac{r}{\varepsilon} - r\right)\exp(-\lambda t) + \begin{cases} Q\ln(r/\varepsilon), & i = 1, \\ 0, & i = 2, \end{cases}$$

$$C_i = (-1)^i Q[1 - \exp(-\lambda T_i)][1 - \exp(-\lambda(T_1 + T_2))]^{-1}, \tag{23}$$

where λ, τ_2, and R_2 are defined in Eq. (12), and $u_3(t, r)$ in Eq. (11).

In this case, the maximum H^+ and minimum H^- values of the borehole yield are

$$H^+ = Q\frac{1 - \exp(-\lambda T_2)}{1 - \exp[-\lambda(T_1 + T_2)]},$$

$$H^- = Q\left[1 - \frac{1 - \exp(-\lambda T_1)}{1 - \exp[-\lambda(T_1 + T_2)]}\right]. \tag{24}$$

It is again simple to estimate Q, p_*, and p_{**} from the easily determined values of T_1, T_2, H^+, and H^-.

Thus, this simple model allows an acceptable qualitative explanation to be given not only for the decrease observed in the rate of pressure drop in the initial period of development of the deposit but also for the appearance of oscillations in the face pressure and yield.

Comparison with Experiment

The quantitative correspondence of the proposed theory and actual data is evaluated by means of the results of industrial experiments on the drop in bed pressure at boreholes 27 and 28 of the Great-Salym field in western Siberia. These data have been used in most of the works cited above and also in [15]. It is assumed that sharp decrease in the rates of pressure drop in the first few months of borehole operation occurs on account of eruption of oil from adjacent lenses.

The dimensional yield $2\pi jH$ of boreholes 27 and 28 is $2 \cdot 10^{-3}$ and $1.6 \cdot 10^{-3}$ m^3/sec, respectively; the pressure at the face at the moment of change in rate of pressure drop is 36.5 and 32 MPa, respectively. At this time, the total volume of petroleum withdrawn is approximately 10^4 m^3 in both cases; the initial bed pressure is 50 MPa; the borehole radius is 0.15 m. It has been established that the lens radius for the given boreholes with piezoconductivity $\varkappa$ = 0.1-10 m^2/sec and hydroconductivity j = $2\cdot10^{-10}$ to $7\cdot10^{-3}$ m^3/Pa·sec is in the range 200-5000 m, which is in good agreement with the physical concepts adopted regarding the collector structure of the Bazhenovskii formation.

Data on the pressure drop in boreholes 27 and 28 as a function of the volume of petroleum collected [15] are compared in Fig. 2 with the corresponding theoretical curves following from the proposed model. The piezoconductivity in both cases is taken to be 1 m^2/sec. It is evident that, despite the roughness and relative arbitrariness of the model concepts adopted, fair agreement between the experimental and theoretical data is observed, permitting the conclusion that these concepts are fundamentally adequate.

Oscillations have been recorded repeatedly on the curves of pressure drop and yield of boreholes in the Salymsk deposits. Their appearance is also well explained by the given method. Reliable estimation of the parameters Q, p_*, and p_{**} from such data entails continuous recording of the face pressure and the volume of petroleum taken over a long period, without change in borehole operation. As follows from the above analysis of self-oscillation, data obtained from boreholes made in relatively small lenses and operating with small yield are most informative from this viewpoint.

Curves of Pressure Recovery and Indicator Diagrams

In estimating the parameters of petroleum-containing beds from the data of hydraulic tests, curves of pressure recovery (CPR) and indicator diagrams (ID) of the boreholes are useful. Both these problems are now briefly considered for operating conditions with a fixed yield, taking β = 0 for the sake of simplicity.

In the problem of finding the CPR, it is assumed, so as to be specific, that the borehole is switched off in conditions when there is constant maintenance of the lens from the surrounding medium (i.e., for H > Q, when there is no self-oscillation). Taking the instant at which the borehole is switched off as the time origin, the initial condition on the pressure field in the lens is obtained on the basis of Eqs. (7)-(10)

$$p(0,\ r) = p^{\circ} + H\ln r - 0{,}5\,(H - Q)(r^2 - 1), \tag{25}$$

where p° is the value of $p_b < p_{**}$ at the time that the borehole is shut off.

The solution of Eq. (1) is sought in the form of a system of functions $p_k(t, r)$, k = 1, ..., determined in successive intervals of dimensionless time of length T_k. The initial condition on $p_1(t, r)$ evidently coincides with Eq. (25) and the initial conditions on $p_k(t, r)$ when $k \geq 2$ take the form $p_k(0, r) = p_{k-1}(T_{k-1}, r)$; T_k is determined from the chain of relations $p_{2k-1}(T_{2k-1}, 1) = p_{**}$, $p_{2k}(T_{2k}, 1) = p_*$. The boundary conditions follow from Eqs. (2) and (3)

$$\lim_{r\to 0} r\frac{\partial p_k}{\partial r} = 0,\quad r\frac{\partial p_k}{\partial r}\bigg|_{r=1} = Q\,\frac{1+(-1)^{k-1}}{2}. \tag{26}$$

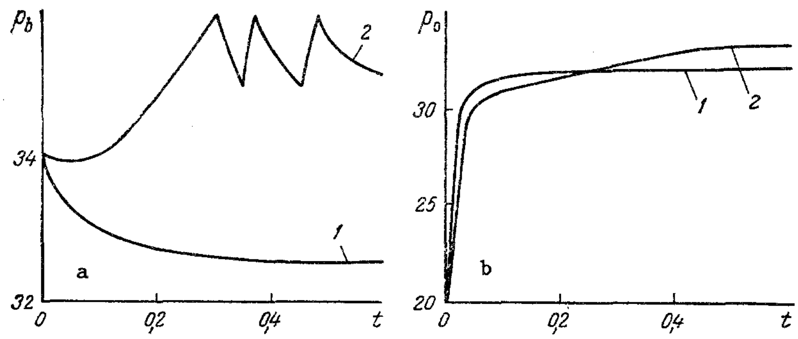

Fig. 3. Influence of external inflow of liquid on the behavior of the pressure at the lens boundary (a) and at the borehole face (b) in the course of pressure recovery at ε = 0.02, p° = 33, p_* = 33.45, p_{**} = 34 MPa, β = 0, H = 3 MPa; 1) Q = 0; 2) 25 MPa.

If this system of functions is found, the pressure in the lens is

$$p(t, r)=\begin{cases}p_1(t, r), & 0\leqslant t<T_1,\\ p_k\left(t-\sum_{i=1}^{k-1}T_i, r\right), & \sum_{i=1}^{k-1}T_i\leqslant t<\sum_{i=1}^{k}T_i.\end{cases} \tag{27}$$

The need to introduce successive intervals of dimensionless time in which the solution of the problem has a different functional form is associated with the possibility that oscillations will appear in the course of pressure recovery. In a definite N-th interval, these oscillations unavoidably cease, i.e., $T_N = \infty$. If oscillations do not appear at all, then obviously N = 2.

Again using the integral-relation method, it is found that

$$p_1(t, r)=p^\circ+\frac{Q}{2}(r^2-1)+2Qt+\begin{cases}u_1(t, r), & R_1(t)\leqslant r\\ u_2(t, r), & R_1(t)>r\end{cases}\Bigg|\ t\leqslant\frac{3}{40},\quad u_3(t-3/40, r),\ t>3/40,$$

$$u_1(t, r)=H\ln r-\frac{H}{2}(r^2-1)-2Ht,\quad R_1(t)=\sqrt{\frac{40}{3}t},$$

$$u_2(t, r)=-\frac{H}{2}(r^2-1)-2Ht+H\left(\ln R_1-\frac{5}{6}+\frac{3}{2}\frac{r^2}{R_1^2}-\frac{2}{3}\frac{r^3}{R_1^3}\right),$$

$$u_3(t, r)=-\frac{H}{4}+H\left(-\frac{7}{30}+r^2-\frac{2}{3}r^3\right)\exp(-9t), \tag{28}$$

$$p_k(t, r)=p_{k-1}(t+T_{k-1}, r)+(-1)^{k-1}u_4(t, r),\ k\geqslant 2,$$

$$u_4(t, r)=\begin{cases}0, & r\leqslant R_2(t)\\ \dfrac{Q(r-R_2)^2}{2(1-R_2)}, & r>R_2(t)\end{cases}\Bigg|\ t\leqslant\frac{1}{8},\quad 0{,}5Qr^2+2Q(t-1/8),\ t>1/8,$$

where $R_2 = 1 - f$ and f is defined in Eq. (10).

Theoretical dependences of the pressure at the lens boundary and at the borehole face on the dimensionless time based on Eqs. (27) and (28) are shown in Fig. 3. The form of the CPR curves obtained for lenses that are hydraulically related with the surrounding medium differs significantly from that for closed lenses (Q = 0). Thus, the model here proposed permits the satisfactory explanation both of a considerable increase in the characteristic time of pressure recovery with the appearance of hydraulic relation of the lenses and of incomplete pressure recovery: the limiting asymptotically achievable pressure in the lens is less than the initial bed pressure by an amount in the range from $p_\infty - p_*$ to $p_\infty - p_{**}$. The presence of liquid influx is able to produce considerable pressure oscillations at the lens boundary, but such oscillations are practically imperceptible at the borehole face.

A series of calculations for the operational conditions of some working boreholes of the Salymsk deposit are performed, in order to investigate the influence of the hydraulic relation between petroleum-containing volumes in collectors with large-scale inhomogeneity on the borehole ID. In such problems, startup of the borehole is with constant yield, but some time after startup the yield changes discontinuously to a new constant value; the pressure field in the lens in this process is found by superposing various of the linear problems on borehole startup considered above (the corresponding solutions are not given here, and likewise there is comparison of theoretical and experimental ID). A general idea of the influence of a hydraulic relation between the lenses on the ID may be obtained on the basis of the curves in Fig. 4, which are plotted for some of the given cases. It is readily evident from an analysis of these curves that the influence of petroleum inflow from the neighboring lenses on the ID form reduces to the appearance of sections that are convex to the depression axis on the ID, which corresponds completely to the ID behavior observed experimentally and in industrial conditions [16]. Note that attempts are fairly often made to explain this behavior in terms of decrease in effective permeability of the

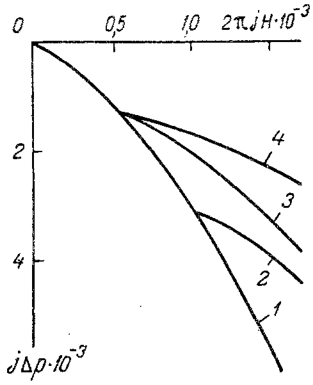

Fig. 4. Characteristic indicator diagrams for a borehole in a lens of radius 200 m; $\kappa = 1$ m²/sec; depression $\Delta p = p_\infty - p_0$, initial yield $2\pi jH = 1.157\cdot10^{-4}$ m³/sec with a discontinuous increase of $2.316\cdot10^{-4}$ m³/sec after each $8.64\cdot10^4$ sec. Curve 1 corresponds to a closed lens, 2 to switching-on of a flow of intensity $1.157\cdot10^{-3}$ m³/sec after a period of $1.054\cdot10^7$ sec from borehole start-up and curves 3 and 4 to switching-on of a flow of intensity $7.52\cdot10^{-4}$ m³/sec after $6.3\cdot10^6$ sec; $\beta = 0.01$ (2, 3) and 0.05 (4); $2\pi jH$, $j\Delta p$, m³/sec.

near-face region of the borehole. This explanation cannot be regarded as correct, since deterioration in hydroconductivity in this region leads, contrariwise, to the appearance of sections that are convex to the yield axis. An example is the filtration to a borehole in a cracked–porous bed, when the pressure reduction in the near-face region leads to partial or complete closure of the cracks, and hence to decrease in permeability there; in this case, the dependence of the yield on the depression reaches a plateau with increase in depression in general [17].

The results obtained reliably indicate that all the anomalous properties of petroleum extraction from significantly inhomogeneous beds of the Bazhenov formation enumerated above may be successfully explained within the framework of a single structural–hydraulic model based on the concept of a hydraulic relation between large-scale inhomogeneities; see also [5, 12]. The conclusions derived from this model are completely acceptable not only in qualitative but sometimes also in quantitative terms, despite the very approximate assumptions regarding the properties of this relation. Therefore, it is expedient to refine this model in the future, directly considering the problem of filtration in two homogeneous lenses immersed in a homogeneous matrix, with a permeability considerably lower than that of the lens material. If there is no limiting gradient with filtration in the matrix, such problems are solved by standard methods using (for circular lenses) a bipolar coordinate system; in the presence of a limiting gradient, the very nontrivial problem of filtration with an unknown boundary is obtained.

NOTATION

b, bed power; H, Q, reduced fluxes equal to the dimensional borehole yield and the flux through the lens boundary divided by $2\pi j$; k, permeability; j, hydroconductivity; p, pressure; p_0, p_b, p_∞, pressure at the face and at the lens boundary, and initial bed pressure; p_*, p_{**}, critical pressure corresponding to the appearance and disappearance of the external flux; R, dimensional radius of lens; R_i, r, dimensionless boundary of filtration zone and radial coordinate; t, T_i, dimensionless time and its characteristic values; u_i, v_i, auxiliary functions; ε, dimensionless borehole radius; $\varkappa$, piezoconductivity; μ, viscosity; τ_i, characteristic intervals of dimensionless time; the superscripts plus and minus denote the maximum and minimum values in the self-oscillatory cycles.

LITERATURE CITED

1. I. I. Nesterov, Geol. Neft. Gaza, No. 10, 26-29 (1979).
2. I. I. Nesterov, Sovetsk. Geol., No. 11, 3-10 (1980).

3. N. D. Kaptelinin, K. S. Yusupov, and V. I. Svezhentsev, in: Geology and Prospecting of Petroleum Deposits in Western Siberia [in Russian], Tyumen' (1978), pp. 96-111.
4. V. M. Dobrynin and V. G. Martynov, Geol. Neft. Gaza, No. 7, 36-43 (1979).
5. I. I. Nesterov, B. P. Stavitskii, A. R. Kurchikov, and A. G. Plavnik, Probl. Neft. Gaza Tyum., No. 44, 15-19 (1979).
6. É. M. Khalimov and V. S. Melik-Pashaev, Geol. Neft. Gaza, No. 6, 1-10 (1980).
7. O. G. Zaripov, V. P. Sonich, and K. S. Yusupov, in: Petroleum Content of Bazhenov Formation of Western Siberia [in Russian], Moscow (1980), pp. 57-67.
8. B. N. P'yankov, Tr. ZapSibNIGNI, No. 180, 71-73 (1982).
9. E. P. Efremov, V. P. Sonich, and V. Sh. Il'in, Neft. Khoz., No. 6, 29-33 (1984).
10. R. I. Medvedskii and K. V. Svetlov, in: Structure and Petroleum and Gas Content of Bazhenov Formation of Western Siberia [in Russian], Tyumen' (1985), pp. 107-111.
11. I. D. Umrikhin, V. K. Fedortsov, O. A. Moskovtsev, et al., in: Industrial and Geophysical Investigation of Bazhenov-Type Petroleum Deposits [in Russian], Tyumen' (1985), pp. 34-43.
12. I. I. Nesterov, B. P. Stavitskii, A. R. Kurchikov, et al., in: Industrial and Geophysical Investigation of Bazhenov-Type Petroleum Deposits [in Russian], Tyumen' (1985), pp. 4-25.
13. I. L. Markhasin, Physicochemical Mechanics of Petroleum Bed [in Russian], Moscow (1977).
14. G. I. Barenblatt, Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 9, 35-49 (1954).
15. V. I. Belkin, E. P. Efremov, and N. D. Kaptelinin, Neft. Khoz., No. 10, 27-31 (1983).
16. V. K. Fedortsov and V. N. Nesterov, Tr. ZapSibNIGNI, No. 127, 17-33 (1978).
17. Yu. A. Buevich and V. S. Nustrov, Inzh.-Fiz. Zh., 48, No. 6, 943-950 (1985).